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# THE MINIMIZATION OF OPEN STACKS PROBLEM

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## KEYWORDS

MOSP, Integer programming, Interval graphs.

## ABSTRACT

The Minimization of Open Stacks Problem is a pattern sequencing problem that is based on the premise that the different items obtained from cutting patterns are piled in stacks in the work area until all items of the same size have been cut. Due to space limitations, it is gainful to find a sequence for the patterns that minimizes the number of open stacks. We have developed an integer programming model based on interval graphs that searches for an appropriate edge completion of the given graph of the problem, while defining a suitable coloring of its vertices.

## INTRODUCTION

The Minimization of Open Stacks Problem (MOSP) is a problem that was first addressed by Yuen in 1991. It arose in the Australian flat glass industry, but it has applications in other cutting industries like steel tubes, paper, wooden panels, as well as in production planning (reducing the space needed for storage of clients' open orders), and also in other fields such as VLSI Circuit Design with the Gate Matrix Layout Problem and PLA Folding, and in classical problems from Graph Theory such as Pathwidth, Modified Cutwidth and Vertex Separation.

The MOSP is a NP-hard problem (Linhares and Yanasse, 2002) which has been addressed in literature by many different methods, such as branch-and-bound techniques (Becceneri et al, 2004), heuristics, genetic algorithms, dynamic programming (Banda and Stuckey, 2007) and integer programming (Baptiste, 2005).

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Consider, for instance, an industry that takes large wooden panels and cuts them in smaller panels of different sizes to meet clients' demands. A specification of how many subpanels of each size will be cut from a larger panel and where the cuts will be made defines a *cutting pattern*. Each cutting pattern can produce different items or just several copies of one same item.

Usually a cutting machine processes just one cutting pattern at a time. The items already cut that are equal are piled in stacks by the machine. The stack of an item type remains near the machine if an item of that type will be cut from a forthcoming pattern. A stack is closed and removed from the work area only after all items of that size have been cut, and immediately before starting to proccess the next cutting pattern. As there are often space limitations around the cutting machines, there is danger of damages on the stacked items, difficulty in distinguishing similar items, and in some cases there is handling costs of removing the stack temporarily to the warehouse, it is advantageous to minimize the number of open stacks, and that can be done simply by finding an optimal sequence to process the cutting patterns.

Most of the authors encounter the minimization of the number of open stacks problem (MOSP) while solving a two stage procedure: first, the classic problem of finding the best patterns to cut stock sheets of glass, paper or wood into smaller rectangular pieces is dealt, usually with an waste minimization purpose, and only after that comes the stage two, which is to determine the sequence in which those patterns should be cut, in order to minimize the number of open stacks.

## MOSP IN A GRAPH

An instance of the MOSP can be associated with a graph having a vertex for each item that is cut and an edge between two vertices if the corresponding items are present in the same cutting pattern (Yanasse, 1997). A pattern with $k$ different items will correspond to a clique of size $k$ in the MOSP graph, because the $k$ vertices must be connected to each other.

The MOSP graph constructed in this way does not show explicitly the cutting patterns, so any model based on it will focus on sequencing the opening of the stacks rather than on sequencing the cutting patterns. That is not a problem, because it is possible to take a solution for the ordering of the vertices of the graph and construct a sequence for the corresponding cutting patterns.

### Interval Graphs

There is a special type of graphs called *interval graphs* (Golumbic, 1980), that are undirected graphs such that its vertices can be put into a one-to-one correspondence with a set of intervals $I$ of a linearly ordered set (like the real line) such that two vertices are connected by an edge if their corresponding intervals intersect. $I$ is called an interval representation for the interval graph.

By associating each open stack of a MOSP problem to an interval in the real line (the interval of time that the stack stays open), we can associate a solution of the MOSP to an interval representation of an interval graph.

## THE MODEL

We have developed an integer programming formulation for the MOSP (Lopes and Carvalho, 2010) based on properties of interval graphs. Given an instance of the problem, we first build the MOSP graph $G=(V,E)$ as explained previously. This model consists in finding out which edges should be added to the original MOSP graph $G=(V,E)$ in order to get an interval graph $H=(V,E\cup F)$ that minimizes the maximum number of simultaneously open stacks.

According to the sequence in which the patterns are proccessed, there may be more or less open stacks simultaneously. Each arc of the future interval graph means that, for a period of time, the two stacks (the respective vertices of the arc) will remain both open. The initial graph contains only the arcs that must be there, in any possible sequence in which the patterns can be proccessed. The rest of the arcs that are added later to the graph will differ according to the sequence of the patterns. It is the choice of these arcs that defines which are the other simultaneously open stacks.

Our model is based on inequalities from a known IP model for the linear ordering problem, and new inequalities inspired on a characterization of interval graphs that uses a linear ordering of the vertices (Olariu, 1991) that corresponds to an ordering of the maximal cliques. The value of the optimum of the MOSP is equal to the size of the biggest clique in the solution graph $H$ and, because interval graphs are perfect graphs, it is equal to the chromatic number of the graph, which is the number of colors needed to assign to the vertices of the graph such that there are no two adjacent vertices of the same color. We also use in the model inequalities derived from the representatives formulation for the vertex coloring problem (Campêlo et al, 2004) to count the number of different colors and find the number of simultaneously open stacks.

## CONCLUSIONS

The integer programming model that was developed for the MOSP has already shown some promising results; furthermore it is being strengthened with symmetry breaking inequalities and lower bounds. This formulation has also been applied to similar problems in pattern sequencing and in graph theory.

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